Chapter 17: Using Mixed Integer Linear Programming (Routing and Wavelength Assignment)

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ECLiPSe ELearning Overview

Licence

Outline

What we want to introduce

- Mixed Integer Linear Programming in ECLiPSe
- `eplex` Libary
- Alternative Models for Routing and Wavelength Assignment in Optical Networks

Problem Definition

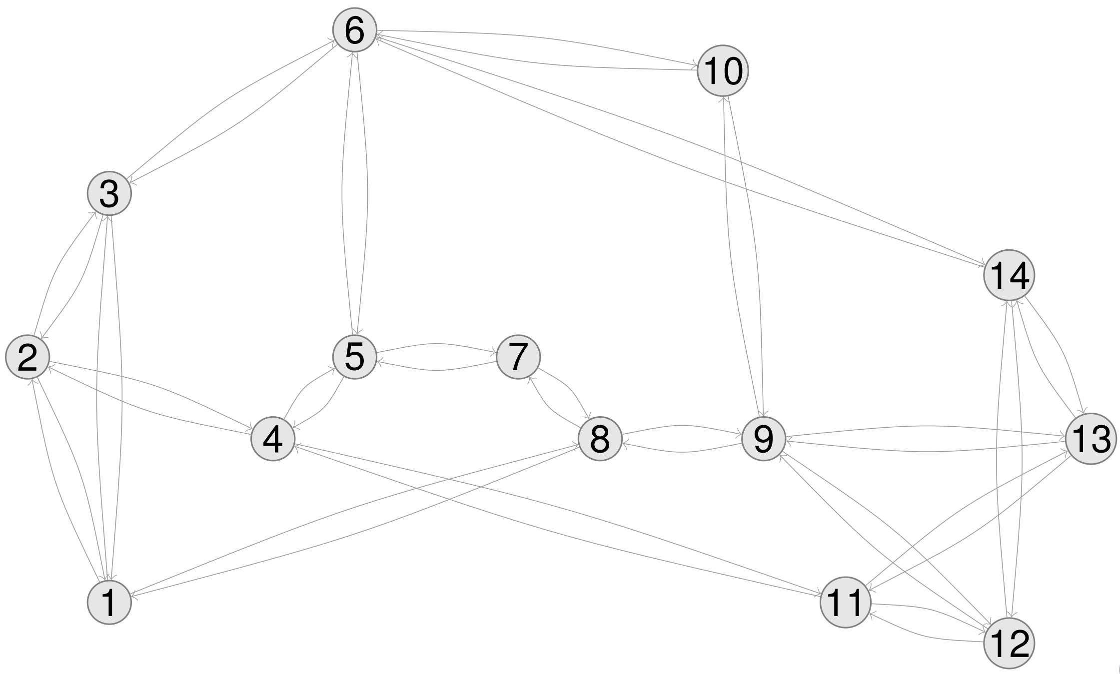
Routing and Wavelength Assignment (Demand Acceptance)

In an optical network, traffic demands between nodes are assigned to a route through the network and a specific wavelength. The route (called *lightpath*) must be a simple path from source to destination. Demands which are routed over the same link must be allocated to different wavelengths, but wavelengths may be reused for demands which do not meet. The objective is to find a combined routing and wavelength assignment which maximizes the number of accepted demands.

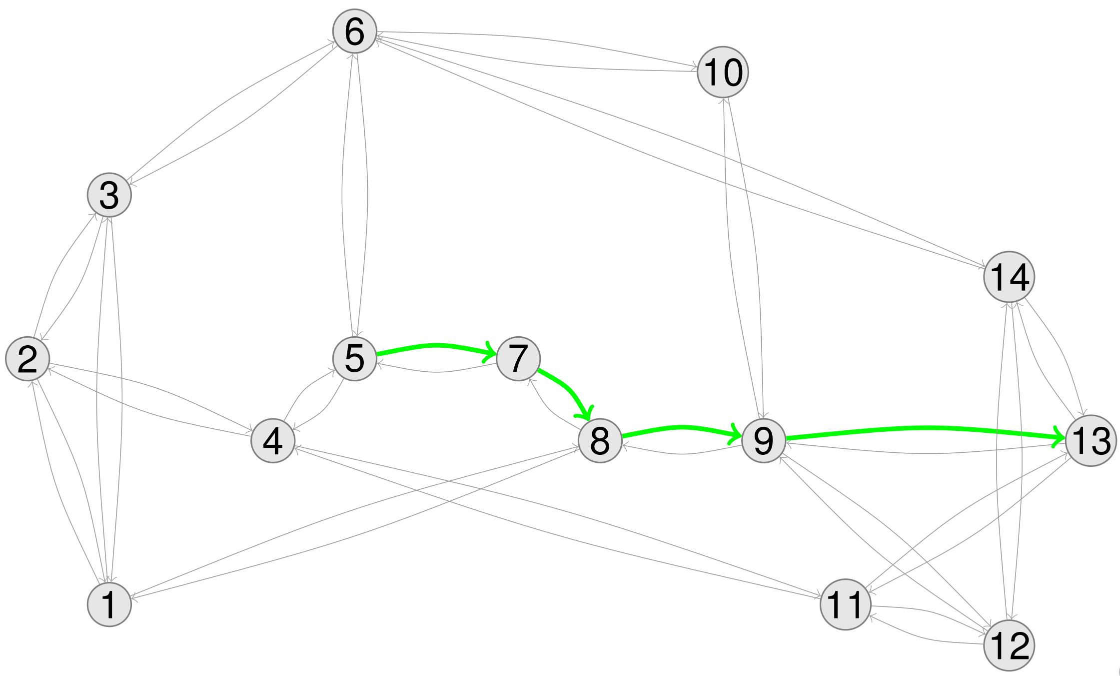
Difference to Previous Problem

- Static problem
  - Accept all demands
  - Minimize number of wavelengths used
  - Design problem, minimize cost of network
- Demand acceptance problem
  - Number of wavelengths fixed
  - Maximize number of accepted demands
  - Operational problem, maximize use of network

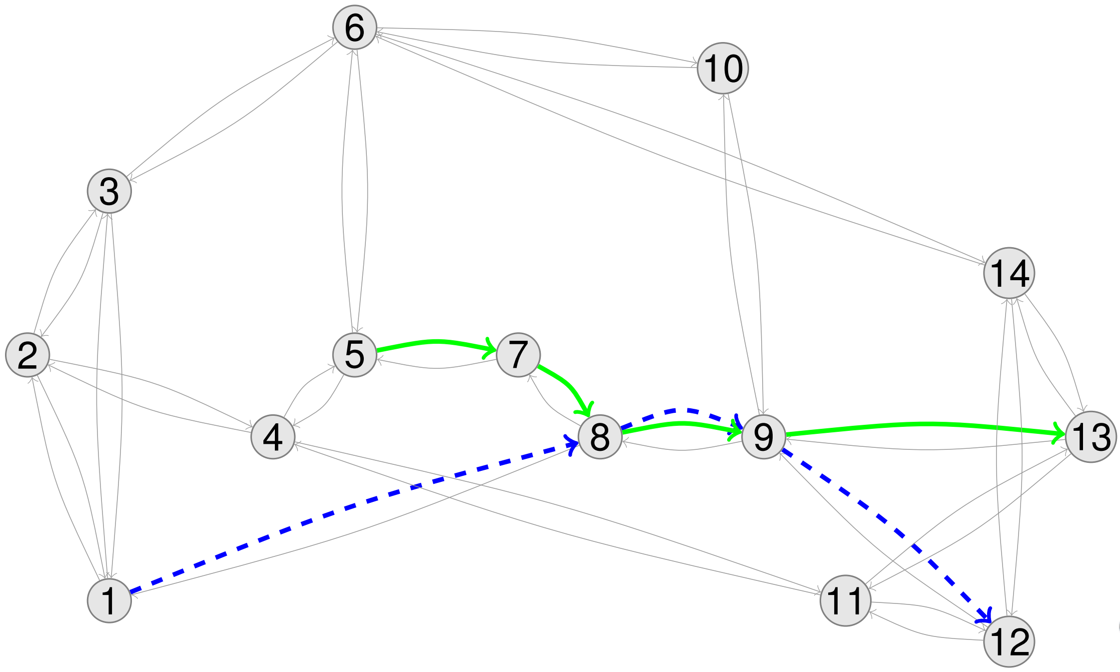
Example Network (NSF, 5 wavelengths)



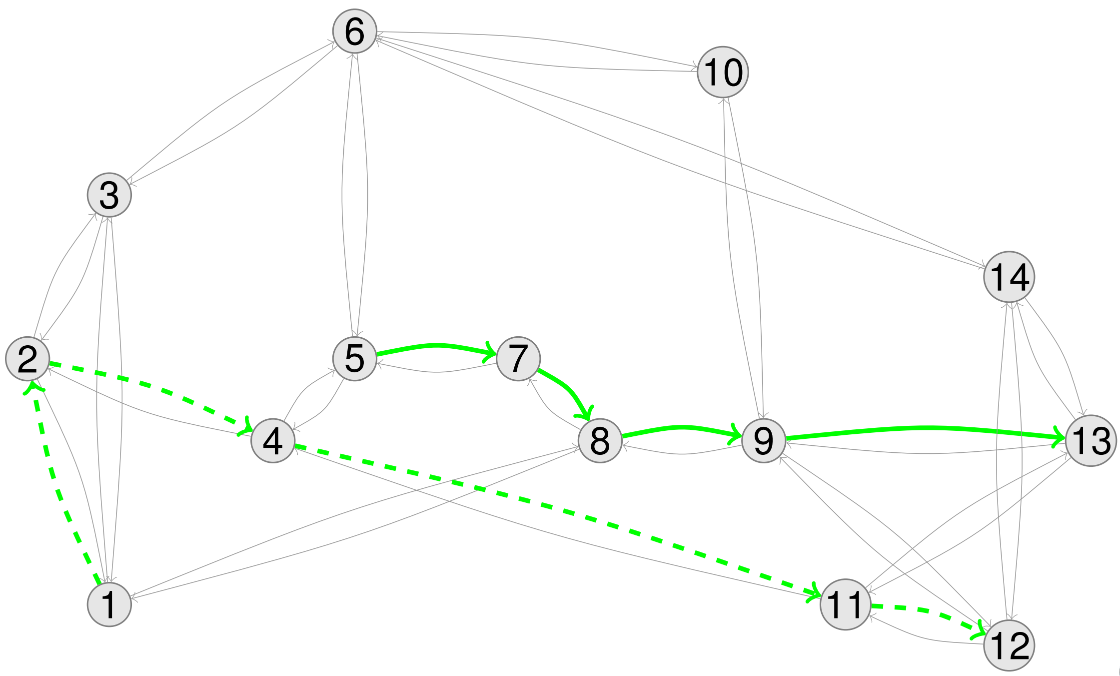
Lightpath from node 5 to node 13 ($5 \Rightarrow 13$)



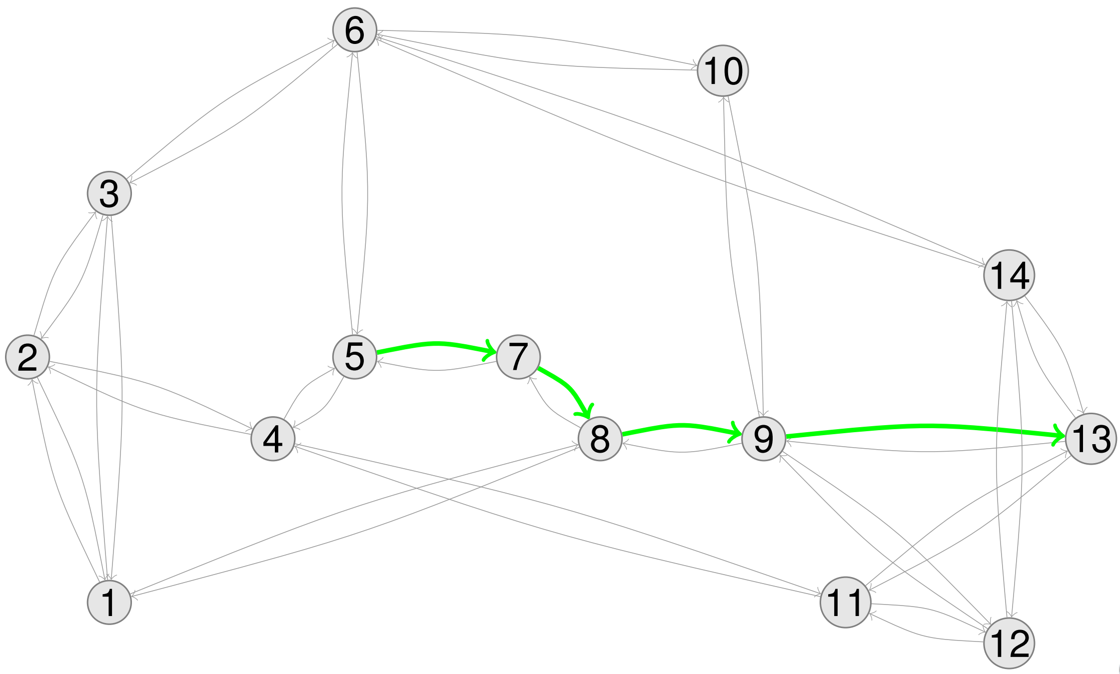
Conflict with demand 1 ⇒ 12: Use different frequencies



Conflict with demand 1 ⇒ 12: Use different path



Conflict with demand 1 $\Rightarrow$ 12: Reject demand



Solution Approaches

- Greedy heuristic
- Optimization algorithm for complete problem
- Decomposition into two problems
  - Find routing
  - Assign wavelengths

Optimization Solutions

- Link Based Model
  - Individual demands
  - Source aggregation
- Path Based Model

Link Based Model - Individual Demands

- Decide for each demand whether it is accepted and which wavelength is used
- Zero/One decision variable y_d^λ
- Atmost one wavelength may be used for demand
- Decide for each link and wavelength if it is used for demand
- Zero/One decision variables x_{de}^λ
- Different demands can not use the same wavelength on the same link
- Maxmize the total number of demands accepted

Notation

- Directed graph $G = (N, E)$
- Nodes $n \in N$
- Edges $e \in E$
- Given Wavelengths $\lambda \in \Lambda$
- Demands $d \in D$ from source $s(d)$ to sink $t(d)$
- Edges $\mathbf{Out}(n)$ leaving node n
- Edges $\mathbf{In}(n)$ pointing to node n

Model Variables

- All Variables 0/1 Integer
- x_{de}^{λ} wavelength λ on link e are used for demand d
- y_{d}^{λ} wavelength λ is used for demand d

Demand Acceptance Model 1

$$\max \sum_{d \in D} \sum_{\lambda \in \Lambda} y_d^\lambda$$

s.t.

$$y_d^\lambda \in \{0,1\}, x_{de}^\lambda \in \{0,1\}$$

$$\forall d \in D: \quad \sum_{\lambda \in \Lambda} y_d^\lambda \leq 1$$

$$\forall e \in E, \forall \lambda \in \Lambda: \quad \sum_{d \in D} x_{de}^\lambda \leq 1$$

$$\forall d \in D, \forall \lambda \in \Lambda: \quad \sum_{e \in \mathrm{In}(s(d))} x_{de}^\lambda = 0, \quad \sum_{e \in \mathrm{Out}(s(d))} x_{de}^\lambda = y_d^\lambda$$

$$\forall d \in D, \forall \lambda \in \Lambda: \quad \sum_{e \in \mathrm{Out}(t(d))} x_{de}^\lambda = 0, \quad \sum_{e \in \mathrm{In}(t(d))} x_{de}^\lambda = y_d^\lambda$$

$$\forall d \in D, \forall \lambda \in \Lambda, \forall n \in N \setminus \{s(d), t(d)\}: \quad \sum_{e \in \mathrm{In}(n)} x_{de}^\lambda = \sum_{e \in \mathrm{Out}(n)} x_{de}^\lambda$$

Recognize Problem Structure

- Minimize/maximize some linear objective
- While satisfying linear equality/inequality constraints
- 0/1, integer or continuous variables

Solving the problem

- This is a standard MILP problem
- MILP = Mixed Integer Linear Programming
- ECLiPSe provides an interface to such solvers
- `eplex` library
- Works with commercial or open-source MIP/LP solvers

Main `eplex` Features used

- Variable definition: `X :: 0.0 .. 1.0`
- Linear constraints `X+Y $= 1`
- Integrality constraints `integers([X,Y])`
- Solver setup `eplex_solver_setup(min(M))`
- Optimization call `eplex_solve(Cost)`

`eplex` Instances

- We can solve multiple MIP problems at same time
- We therefore need to state which problem we want to affect
- This is done with `eplex` instances
- Works like a module: `route:(X+Y $=1)` adds constraint to instance `route`
- Create, use, cleanup

Why not use finite domain solver?

- For this type of problem, finite domain reasoning is very weak
- Each constraint is treated independently
- Interaction through 0/1 variables is limited
- No concept of minimizing objective function

MILP solver basics

- Considers all constraints together
- In form of continuous relaxation
- Use Simplex algorithm to find optimal solution for relaxation
- Integer solutions found by forcing values to be integral
- By branching and/or by adding constraints (cutting planes)

Benchmarks

- Fixed network structure

nsf	14 nodes, 42 edges
eon	20 nodes, 78 edges
mci	19 nodes, 64 edges
brezil	27 nodes, 140 edges

Selected Results (100 runs)

Network	Dem.	λ	Avg LP	Avg MIP	Max Gap	Avg LP Time	Max LP Time	Avg MIP Time	Max MIP Time
brezil	50	5	50.00	50.00	0.00	1.28	1.34	7.31	8.28
brezil	60	5	60.00	60.00	0.00	1.59	1.67	8.40	10.53
brezil	70	5	69.99	69.99	0.00	1.94	2.05	10.97	13.66
brezil	80	5	79.97	79.97	0.00	2.26	2.52	14.13	19.44
eon	50	5	49.99	49.99	0.00	0.73	0.78	3.41	4.38
eon	60	5	59.95	59.95	0.00	0.89	0.99	4.22	9.56
eon	70	5	69.64	69.64	0.00	1.09	1.41	6.16	17.05
eon	80	5	78.99	78.99	0.00	1.40	1.78	10.45	33.91
mci	50	5	49.77	49.77	0.00	0.58	0.64	2.56	3.64
mci	60	5	59.43	59.43	0.00	0.81	1.11	3.65	6.64
mci	70	5	68.73	68.73	0.00	1.07	1.78	6.29	15.49
mci	80	5	77.29	77.29	0.00	1.65	3.76	11.83	33.38
nsf	50	5	49.86	49.86	0.00	0.43	0.55	1.93	4.52
nsf	60	5	59.14	59.14	0.00	0.75	1.31	3.97	10.05
nsf	70	5	66.70	66.70	0.50	1.48	3.03	8.56	28.14
nsf	80	5	72.67	72.63	0.67	2.78	4.42	14.66	62.55
nsf	90	5	77.07	77.04	0.50	3.89	5.77	15.32	51.00
nsf	100	5	81.26	81.20	0.86	4.81	7.05	20.12	80.81

Idea

- Combine all demands starting same node
- Build distribution tree (graph) rooted in source *s*
- Decide whether link/frequency is used for this distribution graph
- Graphs for different source nodes compete for resources
- Enforce sufficient conditions to extract routes for individual demands

Model Notation

- Constants
  - P_{sd} integer, total number of requested demands from s to d
  - D_s, set of all destination nodes for demands sourced in s
- Variables
  - y_{sd} integer variable, how many demands from s to d are accepted (domain 0 to P_{sd})
  - x^{λ}_{se} 0/1 integer variable, frequency λ on link e is used to transport demands starting in s

Source Aggregation Model

$$\max \sum_{s \in N} \sum_{d \in D_s} y_{sd}$$

s.t.

$$y_{sd} \in \{0, 1 ... P_{sd}\}, x^{\lambda}_{se} \in \{0, 1\}$$

$$\forall e \in E, \forall \lambda \in \Lambda : \quad \sum_{s \in N} x^{\lambda}_{se} \leq 1$$

$$\forall s \in N, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(s)} x^{\lambda}_{se} = 0$$

$$\forall s \in N, \forall d \in D_s, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(d)} x^{\lambda}_{se} \geq \sum_{e \in \mathrm{Out}(d)} x^{\lambda}_{se}$$

$$\forall s \in N, \forall d \in D_s : \quad \sum_{\lambda \in \Lambda} \sum_{e \in \mathrm{In}(d)} x^{\lambda}_{se} = \sum_{\lambda \in \Lambda} \sum_{e \in \mathrm{Out}(d)} x^{\lambda}_{se} + y_{sd}$$

$$\forall s \in N, \forall n \neq s, n \notin D_s, \forall \lambda \in \Lambda : \quad \sum_{e \in \mathrm{In}(n)} x^{\lambda}_{se} = \sum_{e \in \mathrm{Out}(n)} x^{\lambda}_{se}$$

And this helps us how, exactly?

- MIP Solution does not say which demands are accepted
- ...nor how they are routed through the network
- Needs solutions extraction, for each source
- At the same time remove loops from routes

Source Model Results (100 runs)

Network	Dem.	λ	Avg LP	Avg MIP	Max Gap	Avg LP Time	Max LP Time	Avg MIP Time	Max MIP Time
brezil	50	5	50.00	50.00	0.00	0.71	0.77	2.49	5.83
brezil	60	5	60.00	60.00	0.00	0.74	0.80	2.77	7.45
brezil	70	5	69.99	69.99	0.00	0.77	0.84	3.02	8.86
brezil	80	5	79.97	79.97	0.00	0.83	0.95	4.76	10.51
eon	50	5	49.99	49.99	0.00	0.29	0.33	1.00	2.20
eon	60	5	59.95	59.95	0.00	0.31	0.38	1.40	2.94
eon	70	5	69.64	69.64	0.00	0.34	0.42	1.91	4.45
eon	80	5	78.99	78.99	0.00	0.40	0.55	2.90	38.94
mci	50	5	49.77	49.77	0.00	0.24	0.36	0.85	2.13
mci	60	5	59.43	59.43	0.00	0.27	0.38	1.38	2.73
mci	70	5	68.73	68.73	0.00	0.32	0.45	2.08	7.42
mci	80	5	77.29	77.29	0.00	0.42	0.66	2.98	7.66
nsf	50	5	49.86	49.86	0.00	0.13	0.16	0.55	1.23
nsf	60	5	59.14	59.14	0.00	0.17	0.23	0.92	2.55
nsf	70	5	66.70	66.70	0.50	0.22	0.33	1.23	5.97
nsf	80	5	72.67	72.63	0.67	0.29	0.58	1.37	5.42
nsf	90	5	77.07	77.04	0.50	0.33	0.48	5.35	379.00
nsf	100	5	81.26	81.20	0.86	0.35	0.64	1.60	9.91

More Information

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